

Adinkra-Stabilized Hypercube Model: Canonical Algebra, Mapping Semantics, and Controlled Simulations

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Abstract

The Adinkra-Stabilized Hypercube (ASH) Model is an exploratory finite-state framework on $\mathbb{F}_2^9$. This paper gives a canonical, executable construction: a rank-four doubly-even linear $[9, 4, 4]$ transform code; a parity-valid application-state subspace; strict one-bit bounded-distance recovery; an idempotent code-orbit projection; and an $N = 8$ Adinkra/Garden representation obtained from the punctured self-dual $[8, 4, 4]$ code. It also specifies eight deterministic image/video measurements, coordinate-9 parity, temporal hysteresis, bounded branch semantics, reconstruction operators, source-consistency scoring, and pruning. Controlled simulations show that the bell-shaped Hamming-weight histogram is the $\text{Binomial}(9, 1/2)$ marginal of uniform occupancy and is not evidence uniquely caused by ASH transforms. Proven finite mathematics, deterministic computation, and interpretive procedural cosmology are therefore stated separately.

1 Scope

ASH studies the composition of finite binary states, code symmetries, graph quotients, procedural branching, and reproducible media-processing semantics. The present contribution is a rigorous finite model and reference implementation. It is not an empirical confirmation of a cosmological theory.

Earlier versions used a genuine coding-theory scaffold but described it imprecisely: a length-nine code was called self-dual, simulations without a decoder were described as correcting errors, and a generic random-state histogram was presented as ASH-specific convergence. Version 1.1.0 resolves those discrepancies.

2 Nine-bit state space

Let

$$V = \mathbb{F}_2^9.$$

Its $2^9 = 512$ states are the vertices of the hypercube Q_9 . Two states are adjacent when their Hamming distance is one, so each vertex has degree nine. Plane k consists of states of weight k and contains $\binom{9}{k}$ vertices.

Application mapping uses

$$E = \{x \in V : x_9 = x_1 \oplus \cdots \oplus x_8\}.$$

This is the kernel of nonzero total parity, hence $\dim E = 8$ and $|E| = 256$.

Deterministic 3D projection of all 512 Q9 vertices

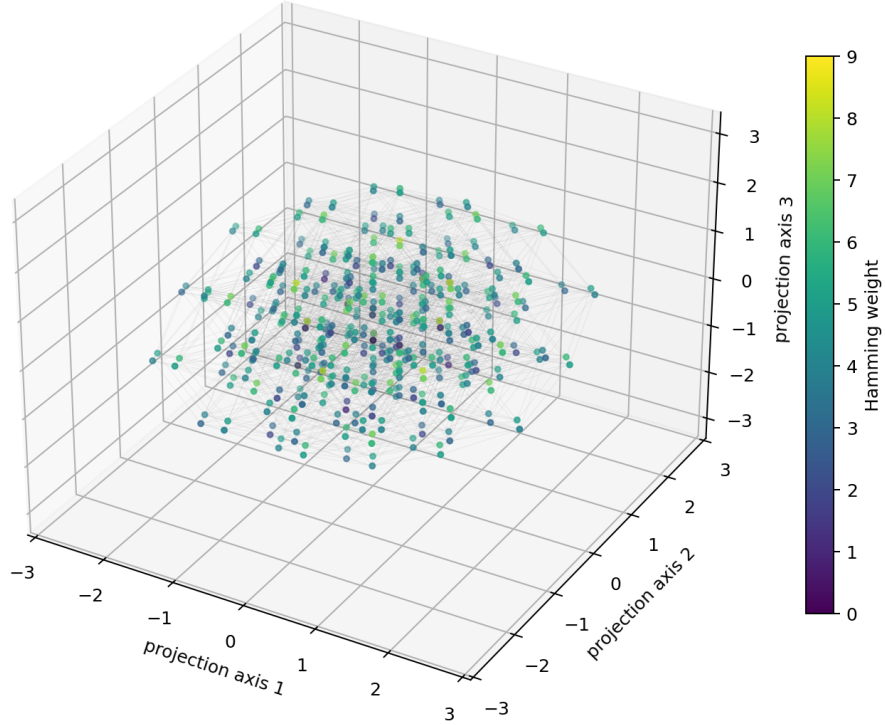


Figure 1: Documented linear visualization of all 512 vertices and all 2,304 edges. It is illustrative, not a physical embedding.

3 Canonical transform code

The code C is the row span over $\mathbb{F}_2$ of

$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Row reduction has four pivots, so $\dim C = 4$ and $|C| = 16$. Exhaustive encoding gives the weight distribution

$$A_0 = 1, \quad A_4 = 14, \quad A_8 = 1.$$

Thus C is doubly even, and its minimum nonzero weight is four. Since the code is linear,

$$d_{\min}(C) = 4.$$

Column 8 of G is zero, so $c_8 = 0$ for every $c \in C$. Each generator satisfies

$$c_9 = c_1 \oplus \cdots \oplus c_8,$$

and linearity extends this relation to the entire code. Coordinate 9 is active because the fourth generator has $c_9 = 1$.

Proposition 1. *C is self-orthogonal but is not self-dual in $\mathbb{F}_2^9$.*

Proof. For $u, v \in C$,

$$\text{wt}(u + v) = \text{wt}(u) + \text{wt}(v) - 2|\text{supp}(u) \cap \text{supp}(v)|.$$

All three weights are divisible by four, so the intersection size is even and $u \cdot v = 0$. Hence $C \subseteq C^\perp$. But $\dim C = 4$ and $\dim C^\perp = 9 - 4 = 5$, so equality is impossible. $\square$

Puncturing coordinate 8 is injective and gives a doubly-even $[8, 4, 4]$ code C_8 . It is self-orthogonal and has half the ambient dimension, so it is self-dual.

4 Strict error recovery

A four-bit message m is encoded as mG . Decoding computes distance to all 16 codewords and succeeds only at distance zero or one.

Theorem 1. *Every one-bit corruption of a codeword is uniquely corrected.*

Proof. Let $d(r, c) \leq 1$. For any other codeword c' , the triangle inequality gives

$$d(r, c') \geq d(c, c') - d(r, c) \geq 4 - 1 = 3.$$

Thus c is the unique nearest codeword. $\square$

The 16 radius-one balls are disjoint and each contains $1 + 9 = 10$ states. The exhaustive decoder partition is 16 exact states, 144 corrected states, and 352 rejected states. All $16 \binom{9}{2} = 576$ two-bit corruptions are rejected; a unique nearest codeword at distance two is not accepted.

For a known affine anchor a , a valid branch state is $a + c$. Decoding $(r + a)$ recovers c through one bit, then reconstructs $a + c$. The implementation checks all 256 integrity-valid anchors, all 16 codewords, and all nine one-bit errors, for 36,864 affine corrections.

Canonical [9,4,4] bounded-distance recovery									
	1	2	3	4	5	6	7	8	9
Codeword	0	1	0	1	1	0	1	0	0
One-bit error	0	1	0	1	1	0	1	0	1
Decoded	0	1	0	1	1	0	1	0	0

status: corrected
distance: 1

status: corrected
distance: 1

Figure 2: Generated nine-bit one-error example and bounded-distance recovery.

5 Code orbits and projection

The additive action of C partitions V into 32 orbits of size 16, and E into 16 such orbits. Coordinate 8 is constant on every orbit.

For $f : V \rightarrow \mathbb{R}$, define

$$(Tf)(x) = \frac{1}{|C|} \sum_{c \in C} f(x + c).$$

Then

$$(T^2f)(x) = \frac{1}{|C|^2} \sum_{a, b \in C} f(x + a + b).$$

For every fixed $d \in C$, exactly $|C|$ pairs satisfy $a + b = d$. Therefore $T^2 = T$. Reindexing also shows that T is self-adjoint and that its image is exactly the space of functions constant on code orbits. Random movement by one codeword is not the same operation as evaluating T ; the implementation exposes exact and Monte Carlo averaging separately.

6 Adinkra quotient and Garden algebra

The quotient Q_8/C_8 has $2^8/2^4 = 16$ vertices. Codeword parity is even, so parity is well-defined on cosets, producing eight bosonic and eight fermionic vertices. Each vertex has one edge of each of eight colors, hence 64 undirected colored edges.

Let

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Define

$$\begin{aligned} L_1 &= I \otimes I \otimes I, & L_2 &= I \otimes I \otimes J, \\ L_3 &= I \otimes J \otimes X, & L_4 &= X \otimes J \otimes Z, \\ L_5 &= Z \otimes J \otimes Z, & L_6 &= J \otimes Z \otimes X, \\ L_7 &= J \otimes I \otimes Z, & L_8 &= J \otimes X \otimes X. \end{aligned}$$

These are signed permutation matrices and, with $R_I = L_I^T$, satisfy exactly

$$L_I R_J + L_J R_I = 2\delta_{IJ} I_8.$$

The executable proof evaluates all 64 matrix pairs with integer residual zero and derives a color-preserving isomorphism with Q_8/C_8 after color permutation $(1, 2, 3, 5, 4, 8, 6, 7)$.

7 Image and video state mapping

Eight measurements define coordinates 1–8: mean luminance, RMS contrast, edge energy, texture residual energy, chroma energy, horizontal-gradient energy, vertical-gradient energy, and temporal change. Every measurement is normalized to $[0, 1]$. Fixed thresholds and hysteresis are normative in `config/ash_mapping_v1.json`. Coordinate 9 is recomputed as parity.

Without history, ties at a threshold map to one. With history, a zero bit switches on at threshold plus hysteresis and a one bit switches off below threshold minus hysteresis. This defines a deterministic temporal transition and rejects an invalid prior state.

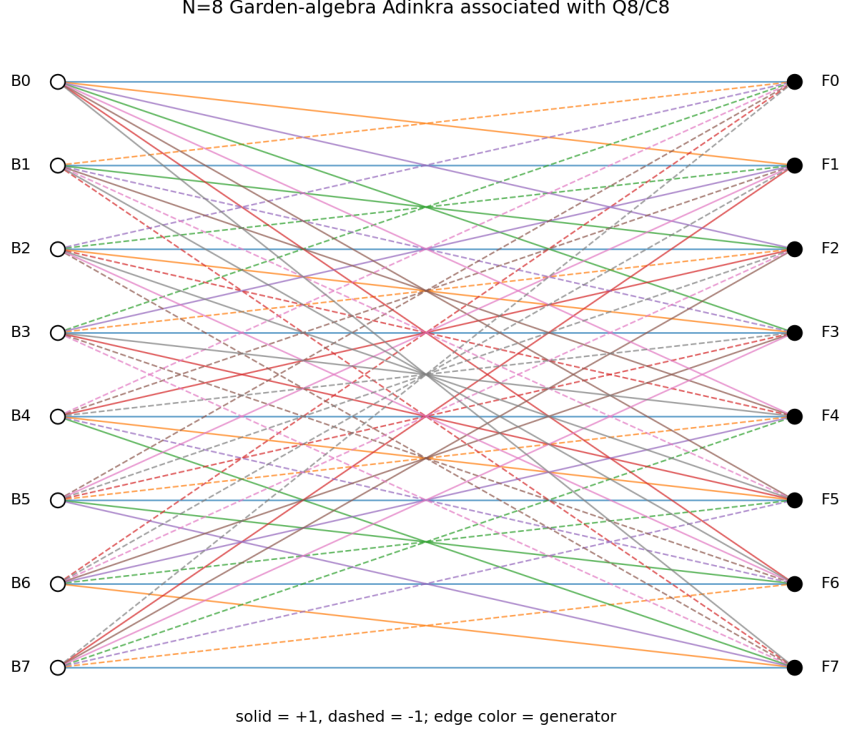


Figure 3: Executable $N = 8$ Garden representation. Dashed edges denote negative entries.

8 Branch and reconstruction semantics

Each semantic node has actions $0, +, -$ with weights $0.5, 0.25, 0.25$. At level ℓ , $+$ toggles message bit $\ell \bmod 4$, $-$ toggles bit $(\ell + 1) \bmod 4$, and 0 leaves the message unchanged. A message is encoded to $c \in C$ and the branch state is $x + c$.

A depth- d tree has 3^d leaves and $(3^{d+1} - 1)/2$ total nodes. Child probabilities sum to the parent probability, so total leaf weight is one. Depth four has 81 leaves, 121 nodes, and reaches all 16 operator messages.

The reference operator family combines horizontal, vertical, and isotropic detail injection with isotropic regularization. Candidates are clipped to $[0, 1]$, reprojected to source resolution, scored by source, edge, and optional temporal loss, and deterministically pruned. This is a conformance reference, not an image-quality claim.

9 Controlled Markov dynamics

For $0 < p < 1$, the noise kernel is

$$P = (1 - p)I + \frac{p}{9} \sum_{i=1}^9 F_i,$$

where F_i flips coordinate i . The chain is irreducible because bit flips generate V , aperiodic because $1 - p > 0$, and doubly stochastic because each F_i is a permutation matrix. Hence the unique stationary distribution is uniform.

Depth-4 bounded branch topology (81 leaves, 16 operator states)

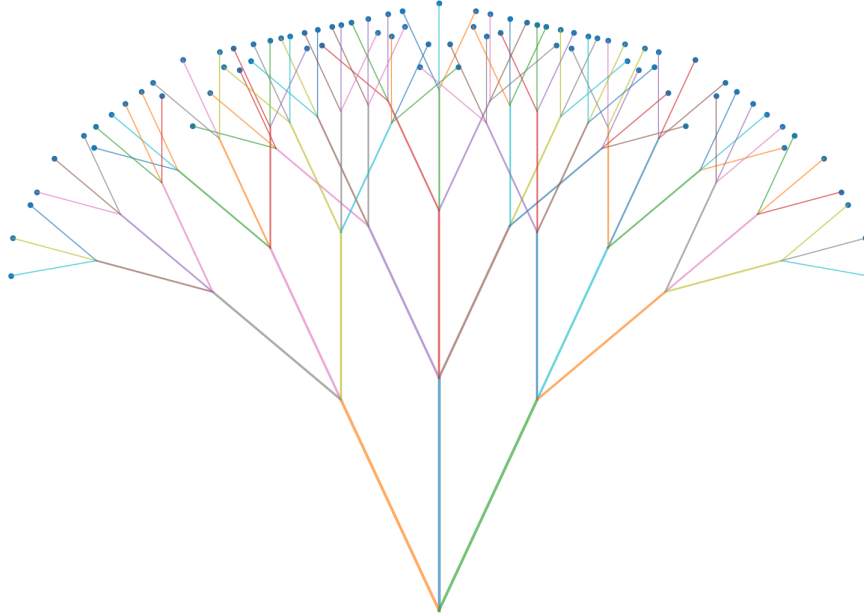


Figure 4: Depth-four bounded branch geometry: 81 leaves and 16 unique operator messages.

Under uniform occupancy, Hamming weight is exactly

$$\Pr(W = k) = \binom{9}{k} 2^{-9}.$$

Codeword translations are also permutations and preserve uniformity. Thus a bell-shaped Hamming histogram is a uniform-state baseline, not ASH-specific evidence.

Without noise, agents starting at zero and translated only by codewords remain in the 16-state code orbit and have weight support $\{0, 4, 8\}$ rather than a full binomial distribution.

10 Verification and limitations

The repository includes exhaustive tests across all 512 states, all one- and two-bit codeword neighborhoods, all parity-valid affine anchors, exact Garden identities, a quotient-graph isomorphism, state and codeword tables, branch topology, controlled ablations, and source/artifact hashes.

The implementation establishes the finite mathematics and deterministic semantics described here. The feature thresholds, branch priors, geometry constants, reconstruction operators, and score weights are versioned reference-design choices; they are not uniquely derived from the interpretive axioms or empirically calibrated as optimal. The implementation does not establish empirical cosmology, a derivation of quantum measurement probabilities, ASH-specific Gaussian causation,

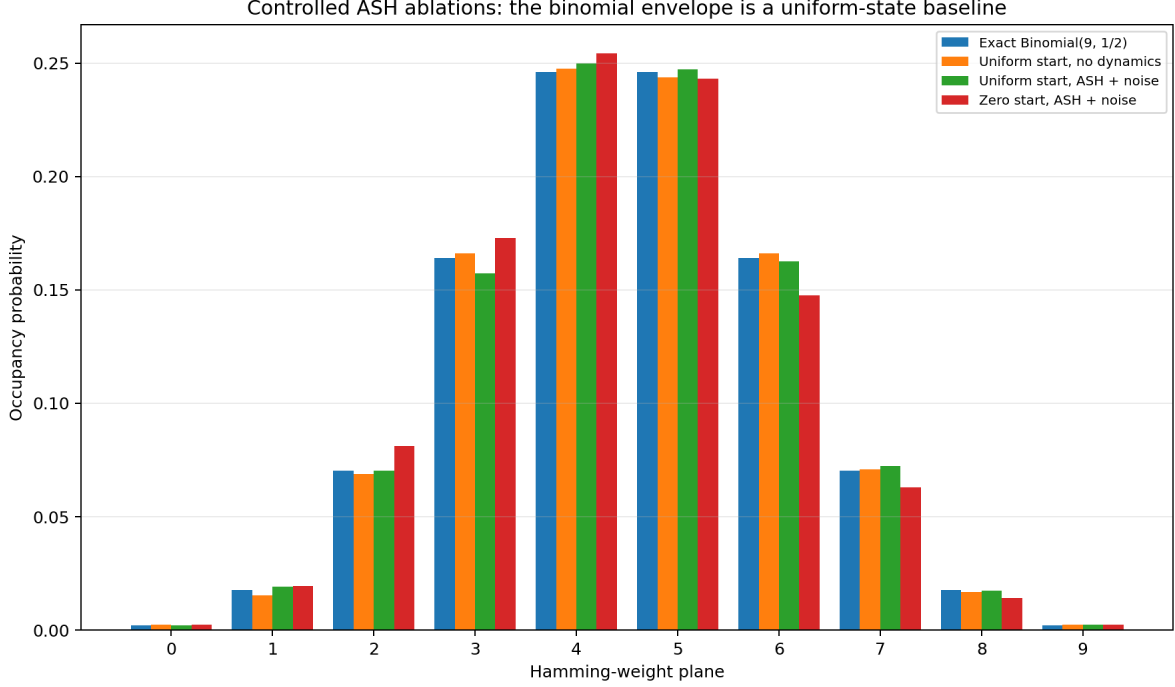


Figure 5: Controlled ablations against the exact binomial envelope.

observed physical supersymmetry, or superior image quality. Those claims require separately designed evidence and matched controls.

11 Conclusion

ASH 1.1.0 is a rigorous finite reference model: a parity-explicit 9-bit state system, a doubly-even $[9, 4, 4]$ code, strict recovery, exact orbit projection, an executable $N = 8$ Adinkra/Garden layer, bounded branch semantics, and deterministic media mapping. Statistical baselines and interpretive limits are now stated explicitly.

References

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